Syntax Analysis

Chapter 4, Part II

Top-Down Parsing

Intro to Top-Down Parsing

- Terminals are seen in order of appearance in the token stream:

  t_1 t_2 t_3 t_4 t_5

- The parse tree is constructed
  - From the top
  - From left to right

Recursive Descent Parsing

- Consider the grammar

  E → T + E | T

  T → (E) | int | int * T

- Token stream is: int * int
- Start with top-level non-terminal E
- Try the rules for E in order

Recursive Descent Parsing. Example (Cont.)

- Try E → T + E
- Then try a rule for T → (E)
  - But (does not match input token int
- Try T → int . Token matches.
  - But + after T does not match input token *
- Try T → int * T
  - This will match but + after T will be unmatched
- Have exhausted the choices for T
  - Backtrack to choice for E

Recursive Descent Parsing. Example (Cont.)

- Try E → T
- Follow same steps as before for T
  - And succeed with T → int * T and T → int
  - With the following parse tree

Recursive Descent Parsing. Notes.

- Easy to implement by hand
  - An example implementation is provided as a supplement “Recursive Descent Parsing”

Recursive-Descent Parsing

- Parsing: given a string of tokens t_1 t_2 ... t_n, find its parse tree
- Recursive-descent parsing: Try all the productions exhaustively
  - At a given moment the fringe of the parse tree is: t_1 t_2 ... t_k A ...
  - Try all the productions for A: if A → BC is a production, the new fringe is t_1 t_2 ... t_k B C ...
  - Backtrack when the fringe doesn't match the string
  - Stop when there are no more non-terminals

When Recursive Descent Does Not Work

- But does not always work ...
- Consider a production S → S a:
  - In the process of parsing S we try the above rule
  - What goes wrong?
- A left-recursive grammar has a non-terminal S

  $S \rightarrow^+ S\alpha$ for some α
- Recursive descent does not work in such cases
  - It goes into a dead loop

Elimination of Left Recursion

- Consider the left-recursive grammar

  $S \rightarrow S\,\alpha \mid \beta$
- S generates all strings starting with a β and followed by a number of α
- Can rewrite using right-recursion

  $S \rightarrow \beta\, S'$

  $S' \rightarrow \alpha\, S' \mid \varepsilon$

Elimination of Left-Recursion. Example

- Consider the grammar

  $S \rightarrow 1 \mid S\,0$ ($\beta = 1$ and $\alpha = 0$)

  can be rewritten as

  $S \rightarrow 1\, S'$

  $S' \rightarrow 0\, S' \mid \varepsilon$

More Elimination of Left-Recursion

- In general

  $S \rightarrow S\,\alpha_1 \mid \ldots \mid S\,\alpha_n \mid \beta_1 \mid \ldots \mid \beta_m$
- All strings derived from S start with one of $\beta_1,\ldots,\beta_m$ and continue with several instances of $\alpha_1,\ldots,\alpha_n$
- Rewrite as

  $S \rightarrow \beta_1\, S' \mid \ldots \mid \beta_m\, S'$

  $S' \rightarrow \alpha_1\, S' \mid \ldots \mid \alpha_n\, S' \mid \varepsilon$

General Left Recursion

- The grammar

  $S \rightarrow A\,\alpha \mid \delta$

  $A \rightarrow S\,\beta$

  is also left-recursive because

  $S \rightarrow^+ S\,\beta\,\alpha$
- This left-recursion can also be eliminated
- See book, Section 4.3 for general algorithm

Summary of Recursive Descent

- Simple and general parsing strategy
  - Left-recursion must be eliminated first
  - … but that can be done automatically
- Unpopular because of backtracking
  - Thought to be too inefficient
- In practice, backtracking is eliminated by restricting the grammar

Predictive Parsers

- Like recursive-descent but parser can "predict" which production to use
  - By looking at the next few tokens
  - No backtracking
- Predictive parsers accept LL(k) grammars
  - L means "left-to-right" scan of input
  - L means "leftmost derivation"
  - k means "predict based on k tokens of lookahead"
- In practice, LL(1) is used

LL(1) Languages

- In recursive-descent, for each non-terminal and input token there may be a choice of production
- LL(1) means that for each non-terminal and token there is only one production that could lead to success
- Can be specified as a 2D table
  - One dimension for current non-terminal to expand
  - One dimension for next token
  - A table entry contains one production

Predictive Parsing and Left Factoring

- Recall the grammar

  E → T + E | T
  T → int | int * T | (E)

- Impossible to predict because
  - For T two productions start with int
  - For E it is not clear how to predict
- A grammar must be left-factored before use for predictive parsing

Left Factoring Rule

- If a production

  $A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \dots \mid \alpha\beta_n \mid \gamma$

  Then

  $A \rightarrow \alpha A' \mid \gamma$

  $A' \rightarrow \beta_1 \mid \beta_2 \mid \dots \mid \beta_n$

Left-Factoring Example

- Recall the grammar

  E → T + E | T
  T → int | int * T | (E)

- Factor out common prefixes of productions

  E → T X
  X → + E | ε
  T → (E) | int Y
  Y → * T | ε

LL(1) Parsing Table Example

- Left-factored grammar
  - E → T X   X → + E | ε
  - T → (E) | int Y   Y → * T | ε
- The LL(1) parsing table:

	int	*	+	(	)	$
T	int Y			(E)		
E	T X			T X		
X			+ E		ε	ε
Y		* T	ε		ε	ε

LL(1) Parsing Table Example (Cont.)

- Consider the [E, int] entry
  - "When current non-terminal is E and next input is int, use production E → T X
  - This production can generate an int in the first place
- Consider the [Y,+] entry
  - "When current non-terminal is Y and current token is +, get rid of Y"
  - We'll see later why this is so

LL(1) Parsing Tables. Errors

- Blank entries indicate error situations
  - Consider the [E,*] entry
  - "There is no way to derive a string starting with * from non-terminal E"

Using Parsing Tables

- Method similar to recursive descent, except
  - For each non-terminal S
  - We look at the next token a
  - And choose the production shown at [S,a]
- We use a stack to keep track of pending non-terminals
- We reject when we encounter an error state
- We accept when we encounter end-of-input

LL(1) Parsing Algorithm

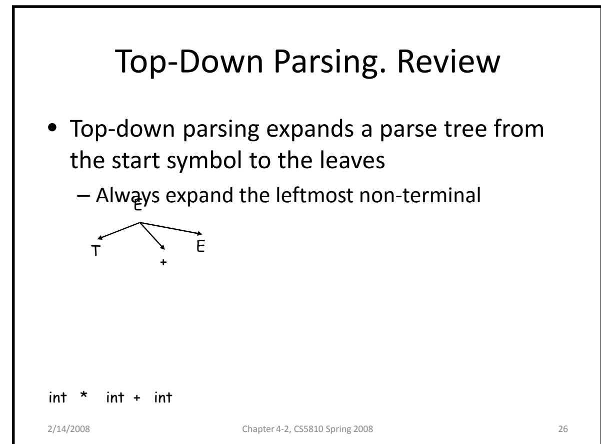
```
initialize stack = <S $> and next (pointer to tokens)
repeat
  case stack of
    <X, rest>  : if T[X,*next] = Y_1...Y_n
                    then stack ← <Y_1... Y_n rest>;
                    else  error ();
    <t, rest>  : if t == *next ++
                    then  stack ← <rest>;
                    else error ();
until stack == < >
```


LL(1) Parsing Example

Stack	Input	Action
E $	int * int $	T X
T X $	int * int $	int Y
int Y X $	int * int $	terminal
Y X $	* int $	* T
* T X $	* int $	terminal
T X $	int $	int Y
int Y X $	int $	terminal
Y X $	$	ε
X $	$	ε
$	$	ACCEPT

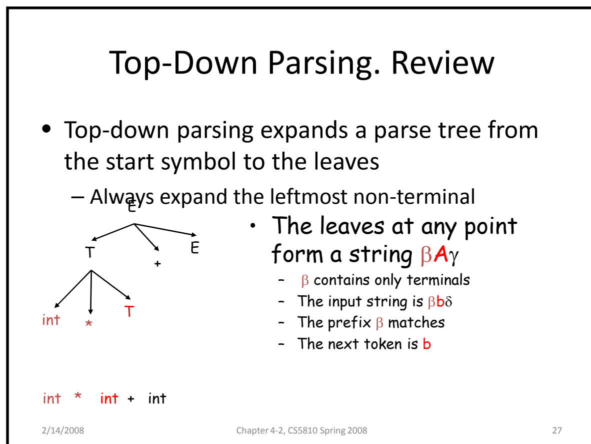
Constructing Parsing Tables

- LL(1) languages are those defined by a parsing table for the LL(1) algorithm
- No table entry can be multiply defined
- We want to generate parsing tables from CFG



Top-Down Parsing. Review

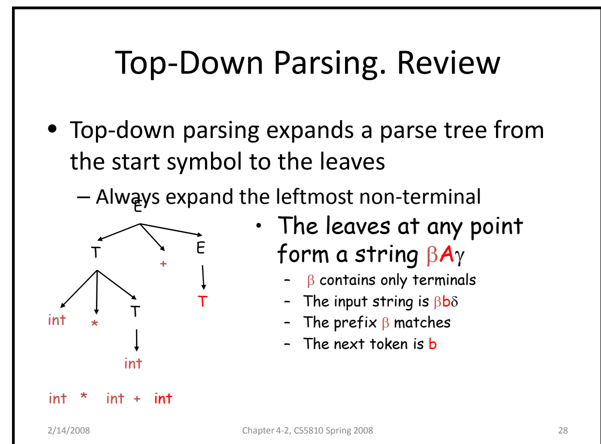
- Top-down parsing expands a parse tree from the start symbol to the leaves
  - Always expand the leftmost non-terminal



Top-Down Parsing. Review

- Top-down parsing expands a parse tree from the start symbol to the leaves
  - Always expand the leftmost non-terminal

- The leaves at any point form a string $\beta A \gamma$
  - β contains only terminals
  - The input string is $\beta b \delta$
  - The prefix β matches
  - The next token is b



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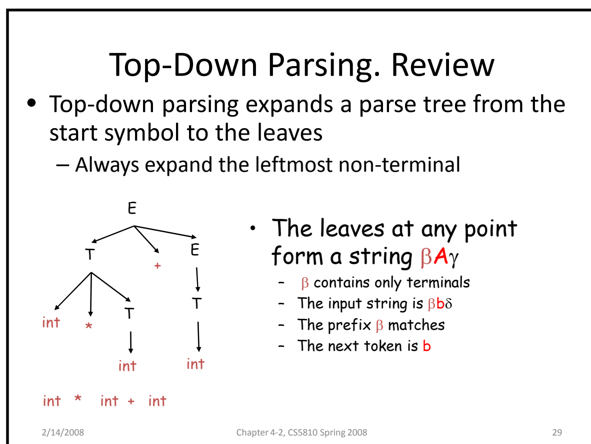

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  - The next token is b

Predictive Parsing. Review.

- A predictive parser is described by a table
  - For each non-terminal A and for each token b we specify a production $A \rightarrow \alpha$
  - When trying to expand A we use $A \rightarrow \alpha$ if b is the next token and A the Non-terminal to be expanded.
- Once we have the table
  - The parsing algorithm is simple and fast
  - No backtracking is necessary

Constructing Predictive Parsing Tables

- Consider the state $S \rightarrow^* \beta A \gamma$
  - With b the next token
  - Trying to match $\beta b \delta$

There are two possibilities:

1. b belongs to an expansion of A
   - Any $A \rightarrow \alpha$ can be used if b can start a string derived from α

   In this case we say that $b \in First(\alpha)$

Or…

Constructing Predictive Parsing Tables (Cont.)

2. b does not belong to an expansion of A
   - The expansion of A is empty and b belongs to an expansion of γ
   - Means that b can appear after A in a derivation of the form $S \rightarrow^* \beta A b \omega$
   - We say that $b \in Follow(A)$ in this case
   - What productions can we use in this case?
     - Any $A \rightarrow \alpha$ can be used if α can expand to ε
     - We say that $\varepsilon \in First(A)$ in this case

Computing First Sets

Definition $First(X) = \{ b \mid X \rightarrow^* b\alpha \} \cup \{ \varepsilon \mid X \rightarrow^* \varepsilon \}$

1. $First(b) = \{ b \}$
2. For all productions $X \rightarrow A_1 \ldots A_n$
   - Add $First(A_1) - \{\varepsilon\}$ to $First(X)$. Stop if $\varepsilon \notin First(A_1)$
   - Add $First(A_2) - \{\varepsilon\}$ to $First(X)$. Stop if $\varepsilon \notin First(A_2)$
   - …
   - Add $First(A_n) - \{\varepsilon\}$ to $First(X)$. Stop if $\varepsilon \notin First(A_n)$
   - Add ε to $First(X)$

First Sets. Example

- Recall the grammar

  $E \rightarrow T X$ $X \rightarrow + E \mid \varepsilon$

  $T \rightarrow (E) \mid int\ Y$ $Y \rightarrow * T \mid \varepsilon$

- First sets

  First(( ) = { (}　First(T) = {int, (}

  First( )) = {) }　First(E) = {int, (}

  First(int) = { int }　First(X) = {+, ε }

  First(+) = { + }　First(Y) = {*, ε }

  First(*) = { * }

Computing Follow Sets

Definition $Follow(X) = \{ b \mid S \rightarrow^* \beta X b \delta \}$

1. Compute the First sets for all non-terminals first
2. Add \$ to Follow(S) (if S is the start non-terminal)
3. For all productions $Y \rightarrow \ldots X A_1 \ldots A_n$
   - Add $First(A_1) - \{\varepsilon\}$ to $Follow(X)$. Stop if $\varepsilon \notin First(A_1)$
   - Add $First(A_2) - \{\varepsilon\}$ to $Follow(X)$. Stop if $\varepsilon \notin First(A_2)$
   - …
   - Add $First(A_n) - \{\varepsilon\}$ to $Follow(X)$. Stop if $\varepsilon \notin First(A_n)$
   - Add $Follow(Y)$ to $Follow(X)$

Follow Sets. Example

- Recall the grammar

  $E \rightarrow T X$ $X \rightarrow + E \mid \varepsilon$

  $T \rightarrow (E) \mid int\ Y$ $Y \rightarrow * T \mid \varepsilon$

- Follow sets

  Follow(+) = { int, (}　Follow(*) = { int, (}

  Follow(( ) = { int, (}　Follow(E) = {), $}

  Follow(X) = {$,) }　Follow(T) = {+,) , $}

  Follow( )) = {+,) , $}　Follow(Y) = {+,) , $}

  Follow(int) = {*, +,) , $}

Constructing LL(1) Parsing Tables

- Construct a parsing table T for CFG G
- For each production $A \to \alpha$ in G do:
  - For each terminal $b \in First(\alpha)$ do
    - $T[A, b] = \alpha$
  - If $\alpha \to^* \varepsilon$, for each $b \in Follow(A)$ do
    - $T[A, b] = \alpha$
  - If $\alpha \to^* \varepsilon$ and $\$ \in Follow(A)$ do
    - $T[A, \$] = \alpha$

Constructing LL(1) Tables. Example

- Recall the grammar

  $E \to T X$ $\quad X \to + E \mid \varepsilon$

  $T \to (E) \mid int\ Y$ $\quad Y \to * T \mid \varepsilon$
- Where in the line of Y we put $Y \to * T$?
  - In the lines of First(*T) = { * }
- Where in the line of Y we put $Y \to \varepsilon$?
  - In the lines of Follow(Y) = { $, +,) }

Notes on LL(1) Parsing Tables

- If any entry is multiply defined then G is not LL(1)
  - If G is ambiguous
  - If G is left recursive
  - If G is not left-factored
  - And in other cases as well
- Most programming language grammars are not LL(1)
- There are tools that build LL(1) tables

Review

- For some grammars there is a simple parsing strategy
  - Predictive parsing
- Next: a more powerful parsing strategy

Exercise

- Consider the following grammar:

  A -> BCD

  B -> **h** B | ε

  C -> C **g** | **g** | C **h** | **i**

  D -> AB | ε
- 1. compute first and follow sets
- 2. give LL(1) parsing table
- 3. show the action of the LL(1) parser on input string hhighh