Using MILP for Optimal Movement Planning in MANETs with Cooperative Mobility

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Abstract— **Rapid-deployment mobile ad-hoc networks (MANETs) are frequently characterized by common over-arching mission objectives which make it reasonable to expect some degree of cooperativeness on the part of their constituent nodes. In this article we demonstrate new strategies to improve MANET communications, based on inter-node cooperation with respect to node mobility. We present our model for cooperative mobility, and use this cost-benefit framework to explore the impact of cooperation in MANETs where nodes are—to varying extents—willing to be moved for the common good. We develop a Mixed-Integer Linear Programming (MILP) formulation of the model, accurately capturing its objectives and constraints. The MILP model is evaluated through simulations and found to be very effective, albeit for small networks. To make the proposed technique scale to large networks we develop a new technique for converting a large global MILP into a sequence of smaller local MILP optimizations, and demonstrate that the resulting approach is scalable and succeeds at efficiently moving cooperative nodes in a manner which optimizes connection bit error rates.**

Index Terms— **wireless ad-hoc networks, bit error rate, cooperative, mixed-integer linear programming.**

I. Introduction

Technical challenges facing MANETs stem from their intrinsic limitations, specifically (i) bandwidth scarcity and high bit error rates of wireless RF channels, and (ii) limited battery capacities which mandate energy-awareness to extend the network lifetime. These limitations have hindered the development of truly scalable QoS-aware routing, and to cope with them much effort has been undertaken to leverage the power of *cooperation* between MANET nodes.

While prior work on the question of how cooperation can benefit communication (e.g. see [6], [4], [3], and others) has principally considered the node's willingness to forward messages as it's cooperative contribution, we explore the ramifications of treating a node's physical mobility as a contributable resource. The assumption, while not applicable in the consumer MANET setting, is quite reasonable in in MANETs where participants have a common (e.g. mission) objective; these rapid deployment settings are precisely the ones where MANETs are most compelling anyway.

II. Cooperative Mobility Model

We consider networks where mobility is a resource that can be used to ameliorate communication infrastructure. Our work begins with the model of Basu et al. [1], but rather than considering networks consisting of robots and non-robots, we consider the more general setting of *heterogenous* networks comprised of nodes which exhibit the entire spectrum of personalities: from defiant autonomy to self-sacrificial cooperativeness. We capture this viewpoint by adopting a cost model for mobility. To wit, every node is willing to move for the sake of the common good, but *for a price*. Each node is assigned a **movement cost** (proportional to distance moved)—this is the price it charges to be moved, say, per meter. Defiant autonomy is exhibited when a node declares this cost to be infinite; self-sacrificial cooperativeness is manifest when this cost is set to zero. The relative extent of cooperativeness exhibited by battlefield MANET nodes is reflected by the ratios of their associated movement costs.

We see mobility planning (for cooperative nodes) as a core function of the network routing layer, which becomes responsible for allocating a fixed (periodically renewed) **mobility budget** towards paying for the movement of cooperative nodes. The model assumes that a node will execute any mobility request that has been adequately funded by an allocation of the mobility budget; such requests are interpreted as being from higher-level supervisors whose objective is to maintain a communication network that best supports the overall mission requirements. Nodes that are autonomous (i.e. unwilling to be subjected to the movement requests of the routing layer) simply declare their movement costs to be infinite.

The central problem to be addressed then is how best to utilize the movement budgets of nodes to defray the cost of for moving them, in a way that leads to meeting the end-to-end QoS requirements of a set of connections. The QoS parameter we consider is bit error rate (BER) as it is gives a good estimate about the quality of the wireless connections. In short, if BER requirements are to be met, which nodes should be moved, and to where?

III. System model

We consider a wireless ad-hoc network consisting of n nodes equipped with omni-directional antennas with different transmition power. Wireless propagation suffers severe attenuation [2] If node i transmits with power P_t(i), the power of the signal received by node j is given by

$$P_{rcv}(j) = \frac{P_t(i)}{c \times d_{ij}^{\alpha}}, \quad (1)$$

where d_{ij} is the distance between nodes i and j. α and c are both constant, and usually $2 \leqslant \alpha \leqslant 4$ (See [2]).

Each wireless channel L between two nodes has a computable Bit Error Rate, $BER(L)$, that is the probability of

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the occurrence of an error during data transfer over that link. The relationship between the BER of a wireless channel and the received power level P_{rcv} is a function of the modulation scheme. Since we are only interested in studying the general dependence of the BER on the received signal power, we will consider the non coherent binary orthogonal Frequency Shift Queying (FSK) modulation scheme. Other modulation schemes can be analyzed in similar way, though closed-form analysis may not be always possible. For this specific modulation scheme, the instantaneous channel BER is given by [7], [8], [5] to be:

$$BER = 0.5\, e^{-\frac{P_{\text{rcv}}}{2P_{\text{noise}}}} \tag{2}$$

Let ρ be a *connection* defined by a source node s and a target node t and consisting of a sequence of links $L_1, \dots, L_r$. In this case, the connection (under an end-to-end retransmission scheme) witnesses

$$BER(\rho) = 1 - \prod_{\ell=1}^{r} 1 - BER(L_\ell). \tag{3}$$

To minimize (3), we maximize $\prod_{\ell=1}^{r} 1 - BER(L_\ell)$, which by monotonicity of logarithms, is equivalent to maximizing $\sum_{\ell=1}^{r} \log(1 - BER(L_\ell))$ or minimizing

$$\sum_{\ell=1}^{r} - \log(1 - BER(L_\ell)). \tag{4}$$

Accordingly, if each link L in a graph is weighted by its *quality*

$$w_{i,j} = - \log(1 - BER(L))$$

then minimum cost paths correspond to minimum routes with minimal end-to-end bit error rates.

In our model, we assign for each cooperative node i a non-negative movement budget b_i. We assume that each node can move to a different location based on the available budget. When a node runs out of budget, it is no longer able change its location.

From the network model, we can see that the quality of the wireless channels within the network can be affected by the location of all nodes. For a given connection, having the many intermediate nodes on the segment between the endpoints would result in lower connection bit error rates; this is deducible from (3). Therefore, our goal(s) are to use the mobility budgets of cooperative nodes to adjust the topology in a manner that:

(I) *meets* end-user connection QoS requirements using minimal node movement, or
(II) *optimizes* end-user connection QoS while not exceeding available movement budgets.

In the rest of the paper, we present our MILP formulation of this online optimization problem, evaluate it through experiments, and propose enhancements to make it scalable to real-world settings.

IV. MILP FORMULATION

Mixed-integer linear programming (MILP) provides a framework for solving optimization problems of this form. In this section, we present our formulation of the optimal mobility planning using MILP.

Assumptions.

(i) The set of mobile nodes $V = \{1, 2, \dots, n\}$ consists of nodes which all have the same sensitivity (minimum receivable signal power P_{min}) and the same transmission power P_{tx}; thus all links in the network are bidirectional.
(ii) The physical environment is discretized by selecting a set of "mesh points" $M = \{1, 2, \dots, N\}$, and declaring that cooperative nodes must be placed only at mesh points'.
(iii) Fast routing convergence occurs and that connections are routed using Dijkstra's shortest path algorithm over the actual topology.

Input

For each node $i \in V$, we are given:

(a) its present *location* $w_i \in M$;
(b) its *mobility budget* $b_i \in \mathbb{R}$;
(c) the *desired BER* $\text{ber}_{i,j}$ between i and $j \in V$.

Preprocessing

- For each pair of positions $p, q \in M$, we compute the channel quality $m_{p,q}$ as $-\log(1 - ber)$, where *ber* is the bit error rate of a direct transmission between locations p and q. These quantities are stored in *channel quality matrix* $[m] = [m_{p,q} \mid p, q \in M]_{N\times N}$.
- For each pair of nodes i and j, we compute the desired QoS $q_{i,j} = -\log(1 - \text{ber}_{i,j})$ and construct the QoS *requirements matrix* $[q] = [q_{i,j} \mid i, j \in V]_{n\times n}$.
- Using pairwise distances, expression (2), and the parameters of assumption (i), we construct the *network graph* $G = (V, E)$. Then, for each pair of nodes i and j, find a shortest route in G from i to j, defining the indicator variable $r_{k,l}^{(i,j)}$ to be 1 iff there is a link from node k to node l ($k, l \in V$) and it was used to route the connection from i to j. These are stored as n^2 distinct *route matrices* $[r]_{i,j} = [r_{k,l}^{(i,j)} \mid k, l \in M]_{N\times N}$ ($i, j \in V$).
- For each node $i \in V$, we compute the distances $d_p^{(i)}$ from w_i to each $p \in M$. These are stored as n distinct *distance vectors* $\vec{d}^{(i)} = [d_p^{(i)} \mid p \in M]_{1\times N}$ ($i \in V$).

Variables

- *Node movement vectors* $\vec{s}^{(i)} = \left[s_p^{(i)} \mid p \in M \right]_{1\times N}$ where $s_p^{(i)} = 1$ iff node i moves to location p (0 otherwise).
- *Link movement variables* are derived from the node movement vectors, where $g_{l,q}^{(k,p)} = 1$ iff nodes k and l are at mesh locations p and q (0 otherwise).

Objective

The actual quality of the connection between nodes i and j (after all nodes have moved) can be computed as

$$X_{i,j} = \sum_{p,q=1}^{N} m_{p,q} \sum_{l,k=1}^{n} r_{k,l}^{(i,j)} g_{l,q}^{(k,p)}. \quad (5)$$

To see this, note that the expression identifies all links (k,l) which are used in connection (i,j) and uses the location p (of k) and q (of l) to determine the quality of each constituent link (k,l); these are then aggregated appropriately to determine the quality of the entire connection (i,j). On the other hand, the distance that node i moves is simply $\vec{d}^{(i)} \cdot (\vec{s}^{(i)})^T$.

We consider an objective function that is a linear combination of these two quantities as sub-objectives:

$$\min \alpha \sum_{i=1}^{n} \vec{d}^{(i)} \cdot (\vec{s}^{(i)})^T + \beta \sum_{i=1}^{n} \sum_{j=1}^{n} X_{i,j}.$$

Here we report on investigations of the pure objective cases, **(Objective I)** when $\alpha = 1, \beta = 0$ and **(Objective II)** when $\alpha = 0, \beta = 1$; in general settings taking both $\alpha, \beta \neq 0$ could be used to implement a mixed objective.

Constraints

- *Movement budget constraints*: To ensure that each node does not violate its movement budget we require that for each $i \in V$:

$$0 \leqslant \vec{d}^{(i)} \cdot (\vec{s}^{(i)})^T \leqslant b_i. \quad (6)$$

- *QoS requirement constraints*: To ensure that the quality of service requirement is met, we require for all $i, j \in V$:

$$X_{i,j} \geqslant q_{i,j}. \quad (7)$$

- *Route constraints*: Since the variables are binary, we require that for all $i, k, l \in V$ and $p, q \in M$:

$$s_p^{(i)}, g_{l,q}^{(k,p)} \in \{0, 1\}. \quad (8)$$

To ensure that node and link movement variables are coherent, we require for all $i \in V$:

$$\left(s_p^{(k)} + s_p^{(l)}\right) - 1 \leqslant g_{l,q}^{(k,p)} \leqslant \min\{s_p^{(k)}, s_p^{(l)}\}. \quad (9)$$

- *Selector constraints*. Since node can only move to one place, for all $i \in V$,

$$\sum_{k=1}^{N} s_p^{(i)} = 1. \quad (10)$$

MILP complexity. The complexity of any MILP problem depends on the number of variables and constraints in that problem. In the proposed formulation, the factors that determine the number of variables and constraints are the mesh size (N), the network size (n), and the connections set C. The number of variables in the proposed MILP formulation is $\sharp vars = (Nn)^2 + Nn$, while the number of constraints involved is $|C| \cdot \sharp vars$.

V. Initial Experiments

We begin by presenting the benefits of cooperative mobility planning using the MILP formulation using the results of some small but very illustrative simulations. The simulations are conducted in $1280m \times 800m$ field. The mesh used was a cartesian grid with cell geometry of $6m \times 6m$. The initial coordinates of the mobile nodes were uniformly randomly distributed within the network. All nodes were given the same transmitting power and the a uniform movement budget (ranging from low values of $50m$ to high values $300m$ in the different experiments). We conducted our optimization on a connection set C of size 3 where the endpoints were chosen at random. We considered both scenarios where the randomly generated connections were edge-disjoint (no contention) and cases where shortest path routes shared edges (contention). We used the MILP solver `lp-solve` which is based on the simplex and branch-and-bound techniques. Figure 1 represents the initial network topology and the set of connections. We analyzed the movement plans determined by MILP in both large $(300m)$ and small $(50m)$ movement budget settings.

Figure 2 shows the new topology output by the MILP solver with Objective I. The connection required BER was met (corresponding to a 60% improvement from the initial value) for all connections while using a total movement budget of 128.

Figure 3 shows the new topology output by the MILP solver with Objective II. The average BER per connection was improved from 10^{-3} to 10^{-9}, an improvement of more than 100%. Under this scheme, a total movement budget of approximately 1000 units was used. For example, for connection C2, a total budget of 568 units was used to lower the end-to-end connection BER from $1.52 \cdot 10^{-3}$ to $2.27 \cdot 10^{-9}$.

In considering the effect of increasing the movement budget on the connection performance we found, as expected, that higher budgets consistently yield better QoS. For example, for connection $C1$, increasing the movement budget from 50 to 300 units, results in lowering BER from $2.45 \cdot 10^{-5}$ to $2.44 \cdot 10^{-7}$.

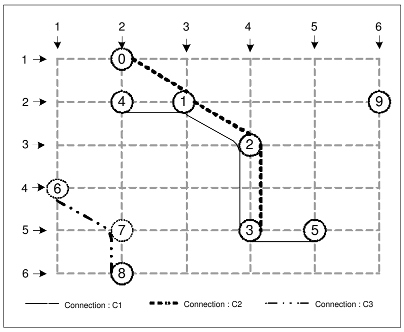

Fig. 1. Initial Topology and connection set

VI. Divide and Conquer

In general, the difficulty of any MILP problem depends on the number of variables and constraints in that problem. We found that MILP problems having more than 2000 variables or

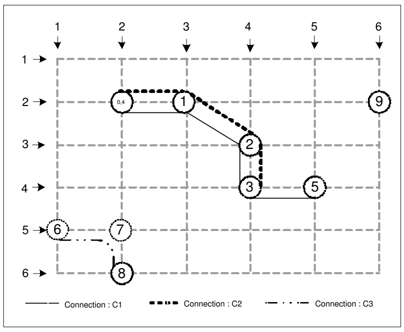

Fig. 2. Objective I, Movement budget = 300/node

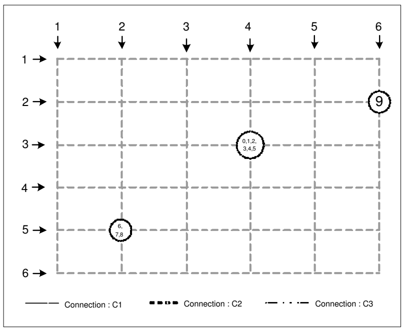

Fig. 3. Objective II, Movement budget = 300/node

4000 constraints were essentially intractable with commodity hardware. Thus, the initial formulation (above) would be helpful only as long as the number of variables and the number of constraints are below these figures. In order to address larger network sizes, we require a strategy for reducing the search space.

Our approach is to replace the global network MILP whose goal is to attain end-to-end connection quality using minimal mobility, by converting it into a set of local MILPs at the link level. This yields scalability by decreasing the computational complexity, providing an improvement of the wireless links, which then indirectly result in an improvement in the end-to-end connection quality. The approach is shown in Figure 5 and is described in detail below:

Each round of the algorithm begins by determining which connections still do not meet their end-to-end bit error rate requirements. For each of these "violating connections" we determine the poorest *improvable* wireless link (s, d), i.e. the link with the highest bit error rate whose endpoints have movement budgets above a predefined threshold.

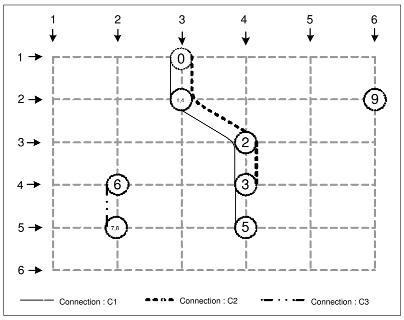

Fig. 4. Objective II, Movement budget = 50/node

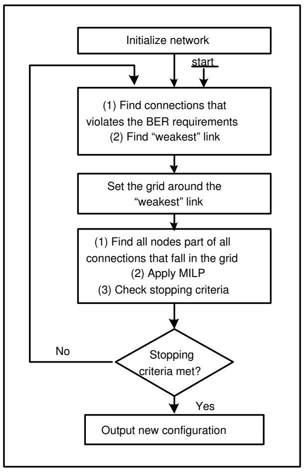

Fig. 5. Converting a global MILP as a sequence of local MILPs

The algorithm then designs a local mesh around link (s, d) by making a uniform cartesian grid around the smallest axis-parallel bounding box which contains s, d and all of their neighbors. The density of the grid is taken to be the same as that for the global MILP, but because the bounding box is typically much smaller than the ambient space, the local MILP involves far fewer mesh points $N' \ll N$.

The procedure then constructs a local MILP using this grid, but considers only those cooperative nodes which both (i) lie within the bounding box and (ii) participate in some connection that does not meet its bit error rate requirements. Because the bounding box is typically much smaller than the ambient space the local MILP considers far fewer nodes $n' \ll n$. Finally, the connection constraints of the local MILP include only the bit error rate requirements of connections going through the link (s, d). The local MILP is solved and node positions updated as prescribed; this completes one round of the algorithm.

The procedure executes additional rounds until convergence; either (i) all connections meet their BER requirements, (ii) the connections which do not meet their requirements contain no improvable links, or (iii) the consideration of all improvable links yields connection BER improvements that are "insignificant", i.e. fall below a chosen threshold.

VII. Results

In this section we give some experimental results to illustrate the performance of the proposed MILP for large network size. The scenario consists of a network size of 25 uniformly distributed nodes, where 10 autonomous nodes are moving according to a Gauss-Markov process, and 15 cooperative nodes operate, each with a uniform mobility budget; all nodes reside inside a one square kilometer grid. Node transmit power and receiver sensitivities are set so that wireless channels arise whenever two nodes are at distance less than $100m$. We establish 15 random connections that we propose BER requirements for the connections equal to 60% of their initial values.

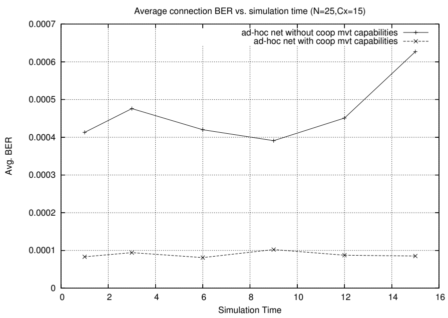

Fig. 6. Using localized MILPs to minize connection BER

The first experiment investigates the impact of the proposed scheme on improving the average BER of the connection set. The top curve in figure 6 represents the average BER of an ad-hoc network where the cooperative nodes remain stationary over time. The bottom curve represents the same measure in the presence of cooperative nodes manipulated according to the proposed MILP scheme. By looking at the *slope* of the bottom chart, we conclude that the routing and optimization scheme were able to maintain a fairly constant low connection set BER. By analyzing the difference of both curves, we conclude that with our proposed scheme, we were able to achieve an improvement of the overall connection set BER by almost 300%.

The second experiment investigates the effects of increasing the node mobility budget (from 20 to 50) and number of connections. By considering the *difference* between the curves curves of the top graph, we notice that for a higher node movement budget corresponds a better improvement in the overall percentage BER improvement. For example, for a connection set size, corresponds a 20% improvement when using 50 units of budget compared to the case where each node has only 20 units. Considering the *slope* of the curves in the top graph, we conclude that the average percentage BER improvement decreases as the connection set size increases.

The bottom graph of figure 7, illustrates the impact increasing the movement budget on the percentage of the connections that do not meet the BER requirement by the time the optimization terminates. By looking at the *slopes*, we conclude that this percentage increases as the connection set size increases. For example, 13% of the connections did not meet the BER requirement when the connection set size equals to 17. By considering the *difference* between both curves of the bottom graph, we conclude the percentage of connections that did not meet the BER requirement is much less in the case of higher movement budget available per node.

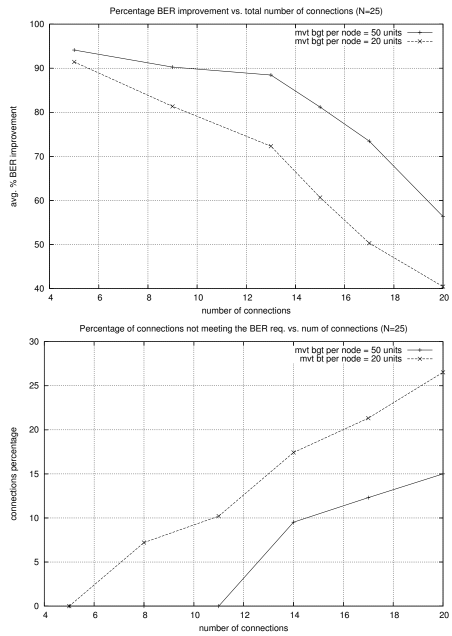

Fig. 7. The benefit of increasing the mobility budget

VIII. Conclusion and Future Work

In this paper, we consider how cooperation between nodes can improve communication in mobile ad-hoc networks (MANETs). We propose a new cooperative mobility model based on location management scheme under budget constraints aiming to the improvement of the QoS of a connection set. We propose an MILP formulation that accurately depicts the proposed cooperative model. Our formal description of this model describes both cases: (1) minimizing the movement budget used by all nodes while meeting the end-to-end QoS requirement of all connections, and (2) minimizing the BER of all connections under movement budget constraints. The MILP model was evaluated through simulations and found to be very effective, albeit for small networks. To make the proposed technique scale to large networks we developed a new technique for converting a large global MILP into a sequence of smaller local MILP optimizations. Simulation experiments indicate conclusively that the resulting approach is scalable and succeeds at efficiently moving cooperative nodes in a manner which optimizes connection bit error rates.

Unfortunately, the current solution formulation is not suitable for a decentralized implementation. In our future work, we will design new distributed schemes for mobility planning, and use the MILP formulation as a baseline by which to assess the relative performance.

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